Absolute Value Equations and Inequalities

Absolute Value Definition - The absolute value of *x*, is defined as…

$$|x| = \begin{cases} x, \ if \ x \geq 0 \\ -x, \ if \ x < 0 \end{cases}$$ where ***x*** is called the "argument"

Steps for Solving *Linear* Absolute Value *Equations*: *i.e.* $|ax + b| = c$

1. Isolate the absolute value.

2. Identify what the isolated absolute value is set equal to…
   a. If the absolute value is set **equal to zero**, remove absolute value symbols & solve the equation to get **one solution**.

   b. If the absolute value is set **equal to a negative** number, there is **no solution.**

   c. If the absolute value is set **equal to a positive** number, set the argument (*expression within the absolute value*) equal to the number **and** set it equal to the opposite of the number, using an 'or' statement in between the two equations. Then solve each equation separately to get **two solutions**.

Examples:

a. $|3x + 12| + 7 = 7$

$|3x + 12| = 0$

Because this equals **0**, there is **ONE** solution.

$3x + 12 = 0$

$3x = -12$

$\boldsymbol{x = -4}$

b. $|3x - 7| + 7 = 2$

$|3x - 7| = -5$

Because this equals a **negative** number, there is **NO** solution.

No Solution

c. $|3x - 7| + 7 = 9$

$|3x - 7| = 2$

Because this equals a **positive** number there are **TWO** sltns.

$3x - 7 = 2$ or $3x - 7 = -2$

$3x = 9$ or $3x = 5$

$\boldsymbol{x = 3}$ or $\boldsymbol{x = \frac{5}{3}}$

d. $|x + 5| = |2x - 1|$ → **Set up two Equations**

$x + 5 = +(2x - 1)$ or $x + 5 = -(2x - 1)$

$\boldsymbol{x = 6}$ or $x + 5 = -2x + 1 \rightarrow$ $3x = -4 \rightarrow$ $\boldsymbol{x = -\frac{4}{3}}$

<u>Steps for Solving *Linear* Absolute Value *Inequalities*:</u> $i.e \quad |ax + b| \le c$

1. Isolate the absolute value.

2. Identify what the absolute value inequality is set "equal" to…

"Zero"

   a. If the absolute value is **less than zero**, there is **no solution.**

   b. If the absolute value is **less than or equal to zero**, there is **one solution**. Just set the argument equal to zero and solve.

   c. If the absolute value is **greater than or equal to zero**, the solution is **all real numbers.**

   d. If the absolute value is **greater than zero**, the solution is all real numbers **except** for the value which makes it equal to zero. This will be written as a **union**.

"Negative" Number

   e. If the absolute value is **less than** <u>or</u> **less than or equal to a negative number,** there is **no solution.** The absolute value of something will *never* be less than or equal to a negative number.

   f. If the absolute value is **greater than** <u>or</u> **greater than or equal to a negative number,** the solution is **all real numbers.** The absolute value of something will *always* be greater than a negative number.

"Positive" Number

   g. If the absolute value is **less than** <u>or</u> **less than or equal to a positive number**, the problem can be approached two ways. Either way, the solution will be written as an **intersection**.

      i. Place the argument in a 3-part inequality (compound) between the opposite of the number and the number, then solve.

      ii. Set the argument less than the number **and** greater than the opposite of the number using an "and" statement in between the two inequalities.

   h. If the absolute value is **greater than** <u>or</u> **greater than or equal to a positive number**, set the argument less than the opposite of the number **and** greater than the number using an 'or' statement in between the two inequalities. Then solve each inequality, writing the solution as a **union** of the two solutions.

3. Graph the answer on a number line <u>and</u> write the answer in interval notation.

Examples:

a.	b.	c.
$\|x - 4\| \ge 0$	$\|2x - 1\| + 4 < 4$ $\|2x - 1\| < 0$	$-3 + \|x + 1\| \le -3$ $\|x + 1\| \le 0$
All Real Numbers	**No Solution**	*Set* $x + 1 = 0$ *So* $\boldsymbol{x = -1}$

d. $|3x+4|+5 \le 3$

$|3x+4| \le -2$

No Solution

e. $2|x-1|-4 \ge 2$

$2|x-1| \ge 6$

$|x-1| \ge 3$

$x-1 \ge 3 \quad OR \quad x-1 \le -3$

$x \ge 4 \quad OR \quad x \le -2$

$(-\infty, -\mathbf{2}] \cup [\mathbf{4}, \infty)$

f. $|x-6|+6 \ge -4$

$|x-6| \ge -10$

All Real Numbers

g. $|2-x| < 8$

$2-x < 8 \quad OR \quad 2-x > -8$

$-x < -6 \quad OR \quad -x > -10$

$x > 6 \quad OR \quad x < 10$

$(\mathbf{6}, \mathbf{10})$

h. $3|4x-1| \le 9$

$|4x-1| \le 3$

i. $|x+6| > 0$

Set $x+6 \ne 0$

So $x \ne -6$

$(-\infty, -\mathbf{6}) \cup (-\mathbf{6}, \infty)$

Problem "h" can be solved using two different approaches.

Option 1 – Split in to two different Inequalities joined by an "AND" statement (Intersection)

$3|4x-1| \le 9$

$|4x-1| \le 3$

$4x-1 \le 3 \quad AND \quad 4x-1 \ge -3$

$x \le 1 \quad AND \quad x \ge -\frac{1}{2}$

$\left[-\frac{\mathbf{1}}{\mathbf{2}}, \mathbf{1}\right]$

Option 2 – Write as a compound inequality (Intersection)

$3|4x-1| \le 9$

$|4x-1| \le 3$

$-3 \le 4x-1 \le 3$ *(add 1)*

$-2 \le 4x \le 4$ *(divide by 4)*

$-\frac{1}{2} \le x \le 1$

$\left[-\frac{\mathbf{1}}{\mathbf{2}}, \mathbf{1}\right]$